The Role of Uncertainty in Model-based Reinforcement Learning

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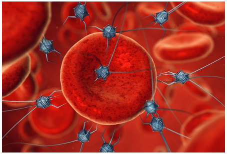
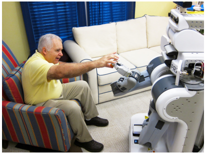
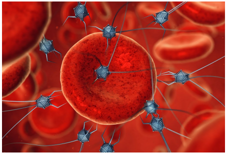
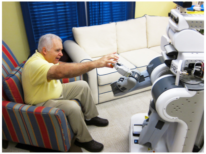
Autonomous Robots

- Vision: Autonomous robots support humans in everyday activities ⏩ Fast learning and automatic adaptation

Autonomous Robots



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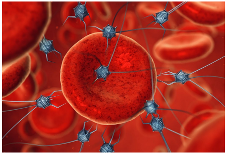

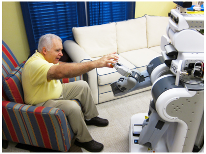


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Fully **autonomous learning and decision making with little data** in real-life situations

Central Problem

Data-Efficient Reinforcement Learning

Ability to learn and make decisions in complex domains without requiring large quantities of data

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▸▸ **Model-based reinforcement learning**

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t, \boldsymbol{u}_t) + \boldsymbol{w}\,, \qquad \boldsymbol{u}_t = \pi(\boldsymbol{x}_t, \boldsymbol{\theta})$$

State

Control

Policy

Policy parameters

Transition function

Reinforcement Learning

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t, \boldsymbol{u}_t) + \boldsymbol{w}\,, \qquad \boldsymbol{u}_t = \pi(\boldsymbol{x}_t, \boldsymbol{\theta})$$

State

Transition function

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Policy parameters

Objective (Controller Learning)

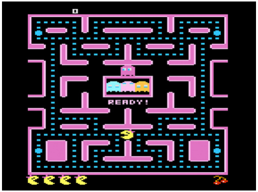
Find policy parameters $\boldsymbol{\theta}^*$ that minimize the expected long-term cost

$$J(\boldsymbol{\theta}) = \sum_{t=1}^{T} \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]\,, \qquad p(\boldsymbol{x}_0) = \mathcal{N}\big(\boldsymbol{\mu}_0,\, \boldsymbol{\Sigma}_0\big)\,.$$

Instantaneous cost $c(\boldsymbol{x}_t)$, e.g., $\|\boldsymbol{x}_t - \boldsymbol{x}_{\text{target}}\|^2$

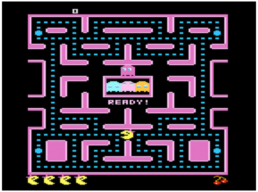
- Typical objective in optimal control and reinforcement learning (Bertsekas, 2005; Sutton & Barto, 1998)

Model-based RL



- Insight: If we had a realistic simulator of the world, we would not need to run experiments in the real world, but in the cheaper simulator (e.g., chess, Go, pacman)

Model-based RL



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- Idea: Build simulator based on observed trajectories

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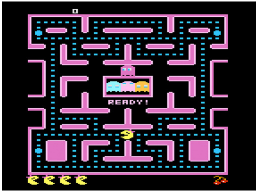

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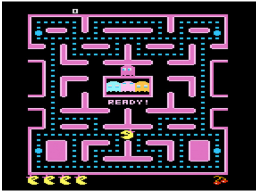

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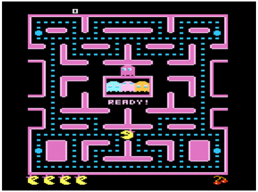

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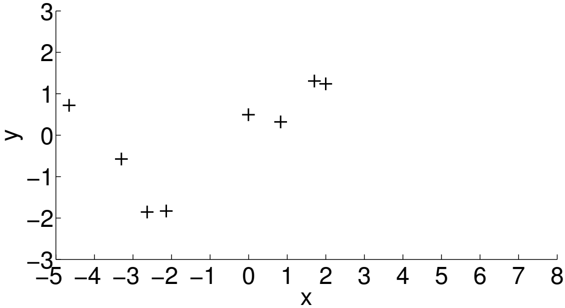
- Idea: Build simulator based on observed trajectories
- Issue: Model errors can lead to catastrophic failures
- Policy is learned only based on simulator data
- How can we build better models? ▶▶ **Probabilistic models**

Model Learning (System Identification)

Model learning problem: Find a function $f : x \mapsto f(x) = y$



Observed function values

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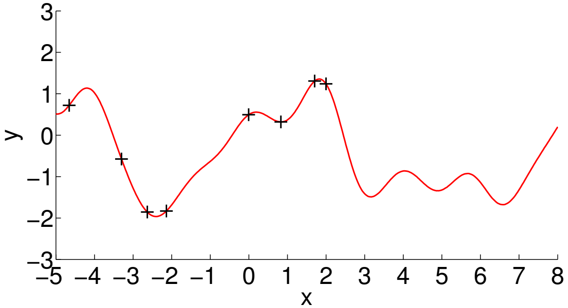

Plausible model

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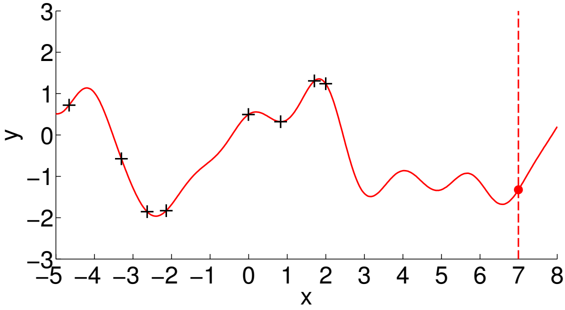

Plausible model

Predictions? Decision Making?

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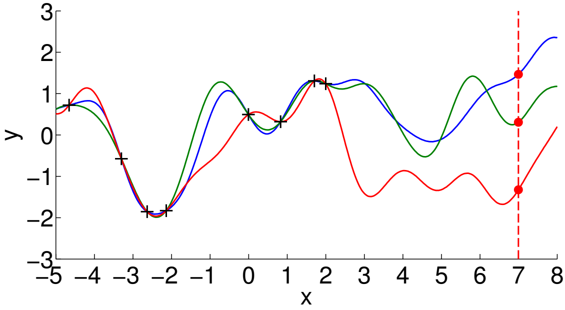

More plausible models

Predictions? Decision Making? Model Errors!

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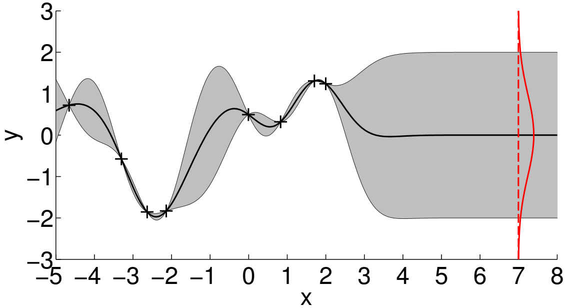

Distribution over plausible functions

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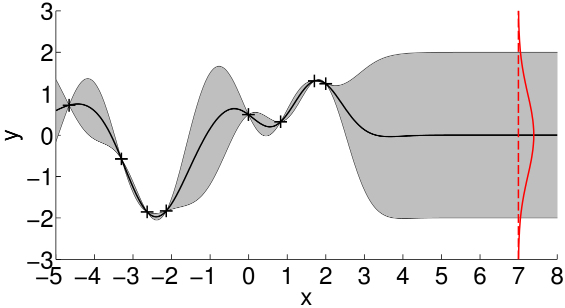

Distribution over plausible functions

- Express uncertainty about the underlying function to be robust to model errors
- Gaussian processes, Bayesian linear regression, ensembles for model learning

Objective

Minimize expected long-term cost $J(\boldsymbol{\theta}) = \sum_t \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]$

PILCO Framework: High-Level Steps

1. Probabilistic model for transition function f
   ▸▸ System identification

Deisenroth et al. (IEEE-TPAMI, 2015): *Gaussian Processes for Data-Efficient Learning in Robotics and Control*

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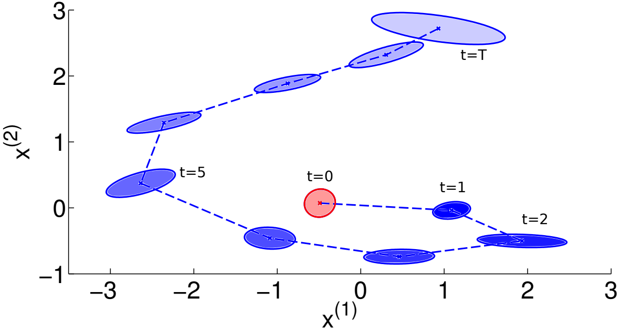
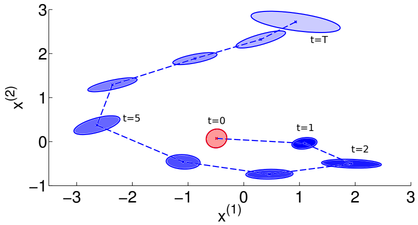
Long-Term Predictions



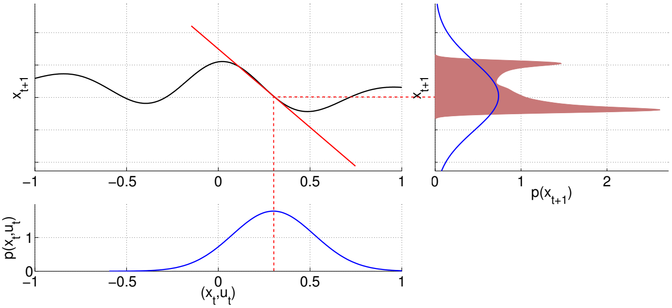
- Uncertainty propagation: Iteratively compute state distributions $p(\boldsymbol{x}_t)$, $t = 1, \ldots, T$.

Uncertainty Propagation



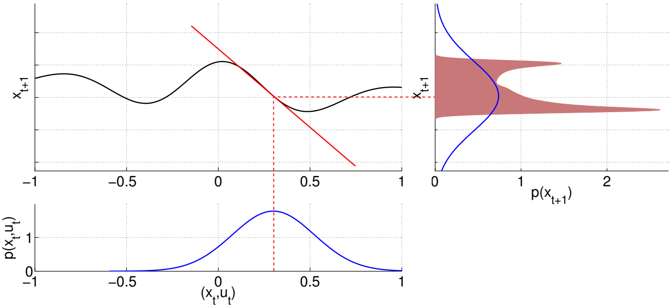
- Deterministic approximate inference (implicit local linearization)
  - Exact moment matching
  - Approximate moment matching (e.g., linearization, unscented transformation)
- Stochastic approximate inference
  - Trajectory sampling
    Important: Compute the GP posterior after every fantasized sample (augment dataset with fantasy data)

Linearization

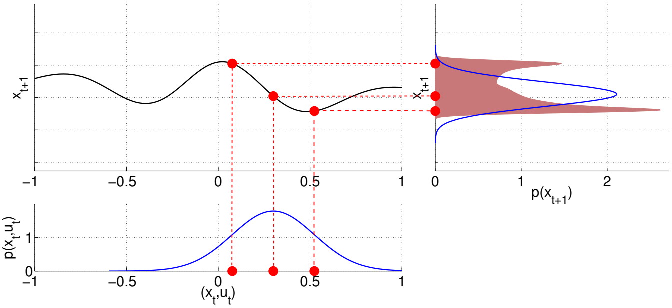


- Approximate nonlinear function at the mean of the distribution with a linear function (1st-order Taylor series)
- Push Gaussian through this linear function: Closed-form mean and covariance of predictive distribution

Linearization

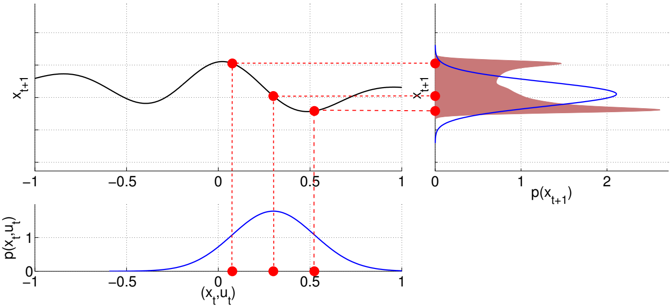


- Approximate nonlinear function at the mean of the distribution with a linear function (1st-order Taylor series)
- Push Gaussian through this linear function: Closed-form mean and covariance of predictive distribution
- Requires gradients of nonlinear function
- Can get the true moments catastrophically wrong



- Approximate distribution with a small number of sigma points
- Evaluate nonlinear function at those points
- Compute sample statistics (mean, covariance) of predictive distribution

Unscented Transformation



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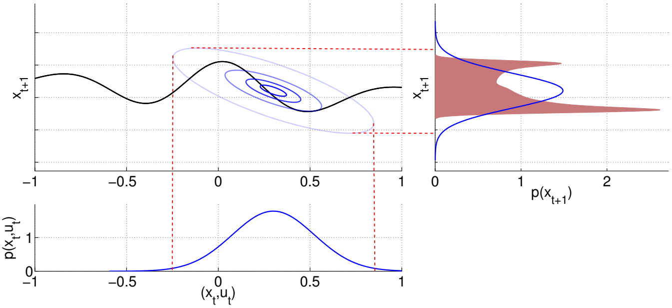
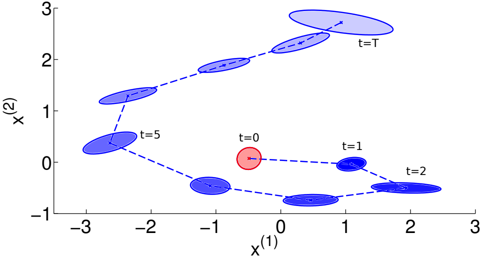
Moment Matching



- Compute mean and covariance of predictive distribution
- Approximate predictive distribution with a Gaussian that possesses the correct moments
- Higher-order moments ignored
- Exact moments can only be computed in special cases



- Iteratively compute $p(\boldsymbol{x}_1|\boldsymbol{\theta}), \ldots, p(\boldsymbol{x}_T|\boldsymbol{\theta})$

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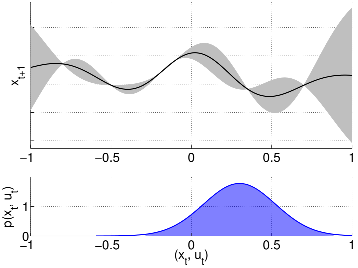
Long-Term Predictions with GPs



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$$\underbrace{p(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t)}_{\text{GP prediction}} \underbrace{p(\boldsymbol{x}_t, \boldsymbol{u}_t|\boldsymbol{\theta})}_{\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})}$$

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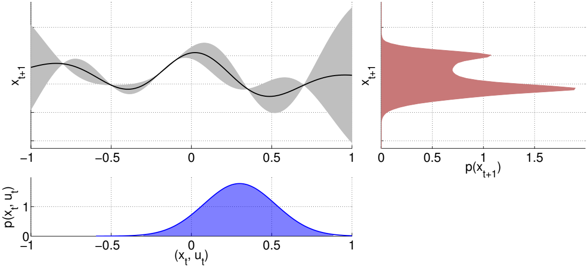

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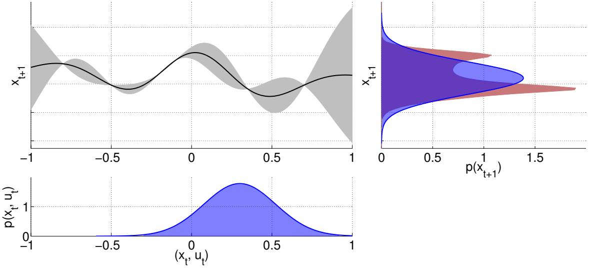

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▶▶ GP moment matching (Girard et al., 2002; Quiñonero-Candela et al., 2003)

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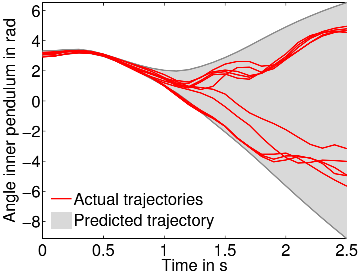
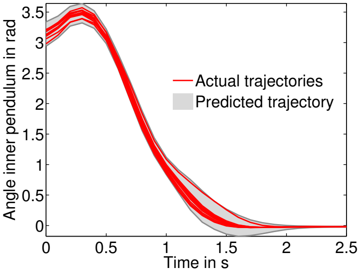
Moment Matching: Long-Term Predictions





- Left: Early stages of learning (model not confident)
- Right: More confident model

▶▶ Predictive error bars seem reasonable

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Minimize expected long-term cost $J(\boldsymbol{\theta}) = \sum_t \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]$

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   - Compute expected long-term cost $J(\boldsymbol{\theta})$
   - Find parameters $\boldsymbol{\theta}$ that minimize $J(\boldsymbol{\theta})$
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Policy Improvement

Objective

Minimize expected long-term cost $J(\boldsymbol{\theta}) = \sum_t \mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}]$

- Know how to predict $p(\boldsymbol{x}_1|\boldsymbol{\theta}), \ldots, p(\boldsymbol{x}_T|\boldsymbol{\theta})$

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- Compute

$$\mathbb{E}[c(\boldsymbol{x}_t)|\boldsymbol{\theta}] = \int c(\boldsymbol{x}_t)\mathcal{N}\big(\boldsymbol{x}_t \,|\, \boldsymbol{\mu}_t,\, \boldsymbol{\Sigma}_t\big) d\boldsymbol{x}_t\,, \quad t = 1, \ldots, T\,,$$

and sum them up to obtain $J(\boldsymbol{\theta})$

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  and sum them up to obtain $J(\boldsymbol{\theta})$
- Analytically compute gradient $\mathrm{d}J(\boldsymbol{\theta})/\mathrm{d}\boldsymbol{\theta}$
- Standard gradient-based optimizer (e.g., BFGS) to find $\boldsymbol{\theta}^*$

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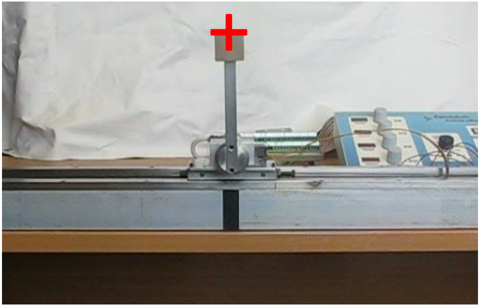
Standard Benchmark: Cart-Pole Swing-up



- Swing up and balance a freely swinging pendulum on a cart
- No knowledge about nonlinear dynamics ⏩ Learn from scratch
- Cost function $c(\boldsymbol{x}) = 1 - \exp(-\|\boldsymbol{x} - \boldsymbol{x}_{\text{target}}\|^2)$

- Code: https://github.com/ICL-SML/pilco-matlab

Deisenroth & Rasmussen (ICML, 2011): *PILCO: A Model-based and Data-efficient Approach to Policy Search*

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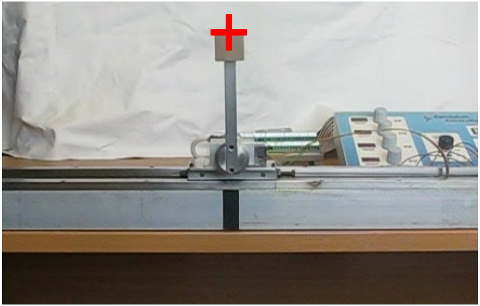

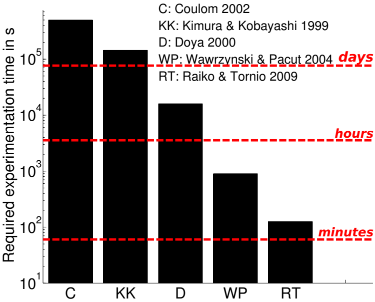

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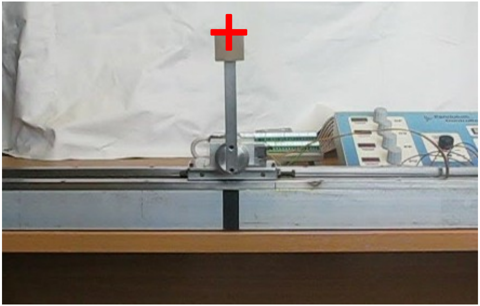

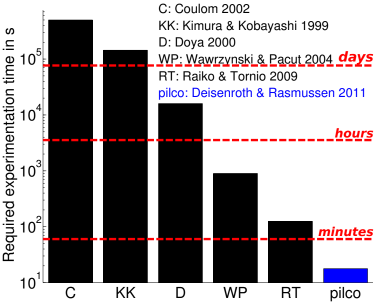

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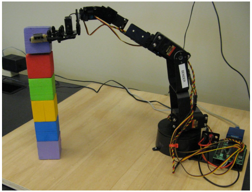
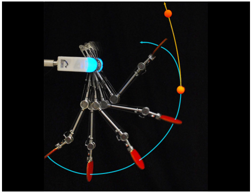
- **Unprecedented learning speed** compared to state-of-the-art
- Code: `https://github.com/ICL-SML/pilco-matlab`

Deisenroth & Rasmussen (ICML, 2011): *PILCO: A Model-based and Data-efficient Approach to Policy Search*

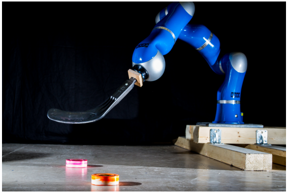
Wide Applicability



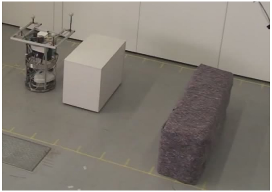
with D Fox



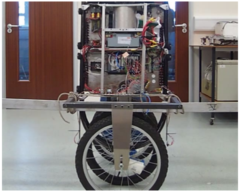
with P Englert, A Paraschos, J Peters



with A Kupcsik, J Peters, G Neumann



B Bischoff (Bosch), ESANN 2013



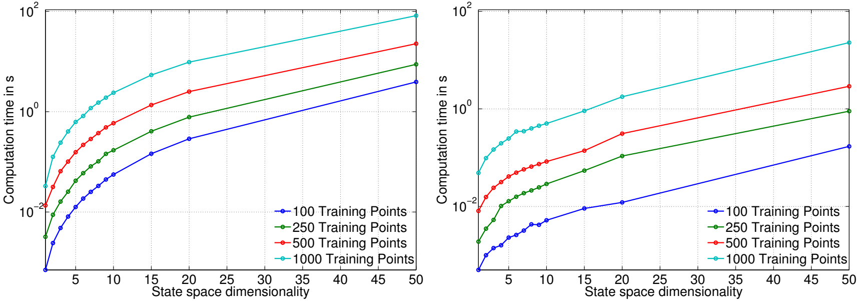
A McHutchon (U Cambridge)

- Application to a wide range of robotic systems

Deisenroth et al. (RSS, 2011): *Learning to Control a Low-Cost Manipulator using Data-efficient Reinforcement Learning*
Englert et al. (ICRA, 2013): *Model-based Imitation Learning by Probabilistic Trajectory Matching*
Deisenroth et al. (ICRA, 2014): *Multi-Task Policy Search for Robotics*
Kupcsik et al. (AIJ, 2017): *Model-based Contextual Policy Search for Data-Efficient Generalization of Robot Skills*

Some More Details

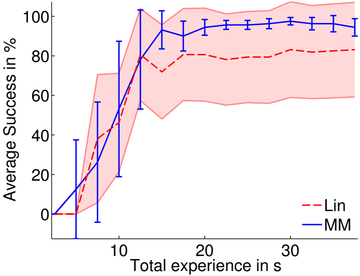
Computational Demand (Inference)



- Graphs: Time required to compute gradient (one time step; 2013 laptop; single CPU)
- Left: Moment matching
- Right: Linearization

▶▶ Linearization is significantly faster than moment matching

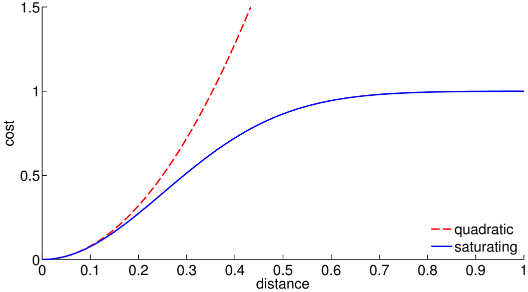
Linearization vs Moment Matching



- These are **NOT** learning curves on training data.
  - Success evaluated on unseen test data
- Linearization is faster (compute), but performs worse than moment matching

Meaningful error bars are useful in RL

Loss Function



- Typical loss function: quadratic
- We use a saturating cost function instead. Why is that?

Quadratic Loss

Assume $\boldsymbol{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $c(\boldsymbol{x}) = \boldsymbol{x}^\top \boldsymbol{x}$. Then

$$\mathbb{E}[c(\boldsymbol{x})] = \mathbb{E}_{\boldsymbol{x}}[\boldsymbol{x}^\top \boldsymbol{x}] = \operatorname{tr}(\boldsymbol{\Sigma}) + \boldsymbol{\mu}^\top \boldsymbol{\mu}$$

- Scales quadratically in the length of $\boldsymbol{\mu}$ (mean deviation from target)
- Scales linearly in the marginal uncertainty of $\boldsymbol{x}$

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- In the early stages of learning, our models are uncertain, and we are far from the target

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- In the early stages of learning, our models are uncertain, and we are far from the target

▶▶ Finding a path to the target may be very costly and even suboptimal (if the planning horizon is not sufficiently large)

Saturating Cost

Assume $\boldsymbol{x} \sim \mathcal{N}\big(\boldsymbol{\mu}, \boldsymbol{\Sigma}\big)$ and $c(\boldsymbol{x}) = 1 - \exp(-\frac{1}{2}\boldsymbol{x}^\top \boldsymbol{x})$. Then

$$\mathbb{E}[c(\boldsymbol{x})] = 1 - \underbrace{|\boldsymbol{I} + \boldsymbol{\Sigma}|^{-\frac{1}{2}}}_{\in[0,1]} \underbrace{\exp(-\tfrac{1}{2}\boldsymbol{\mu}^\top (\boldsymbol{I} + \boldsymbol{\Sigma})^{-1}\boldsymbol{\mu})}_{\in[0,1]} \in [0, 1]$$

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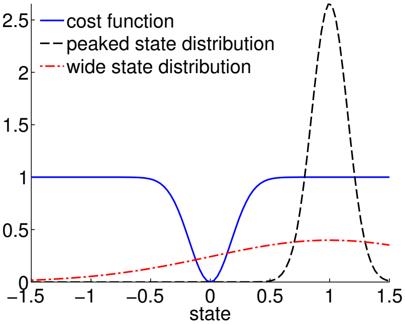
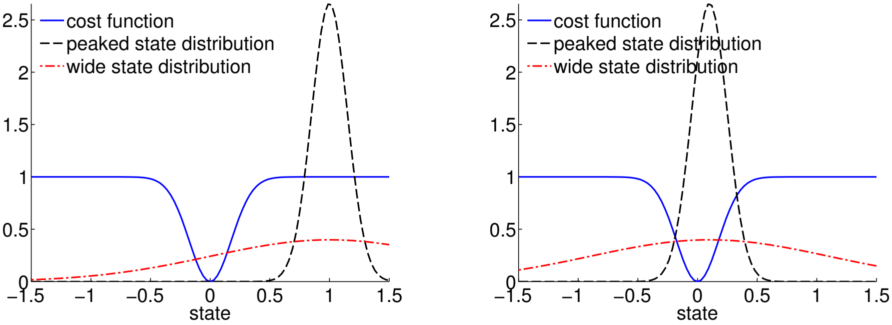
- Large $\boldsymbol{\Sigma}$ can be good (especially if $\boldsymbol{\mu}$ is also large)
- If $\boldsymbol{\mu} \approx \mathbf{0}$ then large $\boldsymbol{\Sigma}$ is not good

Automatic Exploration



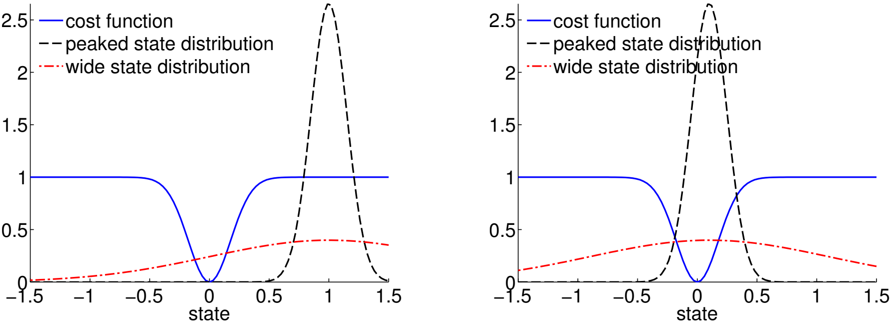
- Far away from the target, uncertainty is favored

Automatic Exploration



- Far away from the target, uncertainty is favored
- Close to the target, uncertainty is discouraged

Automatic Exploration



- Far away from the target, uncertainty is favored
- Close to the target, uncertainty is discouraged

▶▶ Saturating cost automatically deals with the exploration/exploitation trade-off (in some sense)

Probabilistic Model Essential?

DEMO

- Probabilistic model: GP
- Deterministic model: Mean function of GP (still nonparametric)

Deisenroth et al. (IEEE-TPAMI, 2015): *Gaussian Processes for Data-Efficient Learning in Robotics and Control*

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Table: Average learning success with non-parametric transition models

	GP	"Deterministic" GP
Learning success	**94.52%**	**0%**

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Table: Average learning success with non-parametric transition models

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Reasons for failure of deterministic model:

- Model errors: Long-term predictions make absolutely no sense, and the predicted states are nowhere near the target
  ⏩ No gradient signal
- No automatic exploration (model and policy are deterministic)
  ⏩ Stochastic policy fixes this to some degree

Deisenroth et al. (IEEE-TPAMI, 2015): *Gaussian Processes for Data-Efficient Learning in Robotics and Control*

Other Topics

- Predictive error bars are useful for **debugging** (code, model, RL algorithm, ...)
- Learning with **sparse rewards** (get reward only in a tiny area around the goal state) can work, even in continuous state spaces (Deisenroth & Rasmussen, 2011)
- Use predictive uncertainty for **safe exploration** (e.g., Sui et al., 2015; Berkenkamp et al., 2017; Kamthe & Deisenroth, 2018)
- Use BO-type acquisition functions for **exploration incentives** (McAllister 2017)
- **Meta reinforcement learning** (Sæmundsson et al., 2018)
- ...

Wrap-up

- Data-efficient RL is often critical when working with real-world systems
- Model-based RL is one way to accelerate learning
- Key: **Probabilistic models reduce the effect of model errors**
- "Faithful" uncertainty propagation leads to faster learning
- Classical quadratic losses are not good when working with uncertainty
- Saturating loss allows for automatic exploration/exploitation

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