Integration

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Integration problems

- Moment computation

$$M_k(x) = \int x^k p(x)dx$$

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- Evidence (marginal likelihood)

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- Evidence (marginal likelihood)

$$p(X) = \int p(X|\theta)p(\theta)d\theta$$

- Relative entropy (KL divergence)

$$\mathrm{KL}(p||q) = \int \log \frac{p(x)}{q(x)} p(x)dx$$

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- Prediction in time-series models

$$p(x_{t+1}) = \int p(x_{t+1} \mid x_t)p(x_t)dx_t$$

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- Experimental design

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$$\mathbb{E}\,[U(x)] = \int U(x; \;)p(\;)d$$

- Planning

$$J\;(x(0)) = \int_0^T r(x(t); u(t)) \mid x(0)dt$$

Exact integration

- Compute integrals analytically, if possible (Gradshteyn & Ryzhik, 2007)

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Approximate integration

Topic	Useful reference	Video
Numerical integration	Stoer & Bulirsch (2002)	Chapter 1
Bayesian quadrature	Rasmussen & Ghahramani (2003), Gunter et al. (2014)	Chapter 1
Monte-Carlo integration	MacKay (2003), Murray (2015)	Chapter 2
Normalizing flows	Weng (2018), Papamakarios et al. (2019), Kobyzev et al. (2020)	Chapter 3
Inference in time series	Julier & Uhlmann (2004), Särkkä (2013)	Chapter 4

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